OCR
Oxford Cambridge and RSA

# Monday 13 May 2019 – Afternoon

## AS Level Further Mathematics A

**Y531/01** Pure Core

**Time allowed: 1 hour 15 minutes**



**You must have:**
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

**1** You are given that $z = 3 - 4\mathrm{i}$.

**(a)** Find

- $|z|$,
- $\arg(z)$,
- $z^*$. **[3]**

On an Argand diagram the complex number $w$ is represented by the point $A$ and $w^*$ is represented by the point $B$.

**(b)** Describe the geometrical relationship between the points $A$ and $B$. **[2]**

**2** Matrices **P** and **Q** are given by $\mathbf{P} = \begin{pmatrix} 1 & k & 0 \\ -2 & 1 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} (1+k) & -1 \end{pmatrix}$ where $k$ is a constant.

Exactly one of statements A and B is true.

Statement A: **P** and **Q** (in that order) are conformable for multiplication.
Statement B: **Q** and **P** (in that order) are conformable for multiplication.

**(a)** State, with a reason, which **one** of A and B is true. **[2]**

**(b)** Find either **PQ** or **QP** in terms of $k$. **[2]**

**3** The position vector of point $A$ is $\mathbf{a} = -9\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$.
The line $l$ passes through $A$ and is perpendicular to **a**.

**(a)** Determine the shortest distance between the origin, $O$, and $l$. **[2]**

$l$ is also perpendicular to the vector **b** where $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$.

**(b)** Find a vector which is perpendicular to both **a** and **b**. **[1]**

**(c)** Write down an equation of $l$ in vector form. **[1]**

$P$ is a point on $l$ such that $PA = 2OA$.

**(d)** Find angle $POA$ giving your answer to 3 significant figures. **[3]**

$C$ is a point whose position vector, **c**, is given by $\mathbf{c} = p\mathbf{a}$ for some constant $p$. The line $m$ passes through $C$ and has equation $\mathbf{r} = \mathbf{c} + \mu\mathbf{b}$. The point with position vector $9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$ lies on $m$.

**(e)** Find the value of $p$. **[3]**

**4 In this question you must show detailed reasoning.**

You are given that $f(z) = 4z^4 - 12z^3 + 41z^2 - 128z + 185$ and that $2 + i$ is a root of the equation $f(z) = 0$.

**(a)** Express $f(z)$ as the product of two quadratic factors with integer coefficients. **[5]**

**(b)** Solve $f(z) = 0$. **[3]**

Two loci on an Argand diagram are defined by $C_1 = \{z : |z| = r_1\}$ and $C_2 = \{z : |z| = r_2\}$ where $r_1 > r_2$. You are given that two of the points representing the roots of $f(z) = 0$ are on $C_1$ and two are on $C_2$. $R$ is the region on the Argand diagram between $C_1$ and $C_2$.

**(c)** Find the exact area of $R$. **[4]**

**(d)** $\omega$ is the sum of all the roots of $f(z) = 0$.

Determine whether or not the point on the Argand diagram which represents $\omega$ lies in $R$. **[2]**

**5 In this question you must show detailed reasoning.**

You are given that $\alpha$, $\beta$ and $\gamma$ are the roots of the equation $5x^3 - 2x^2 + 3x + 1 = 0$.

**(a)** Find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[5]**

**(b)** Find a cubic equation whose roots are $\alpha^2$, $\beta^2$ and $\gamma^2$ giving your answer in the form $ax^3 + bx^2 + cx + d = 0$ where $a$, $b$, $c$ and $d$ are integers. **[4]**

**6** A transformation T is represented by the matrix $\mathbf{T}$ where $\mathbf{T} = \begin{pmatrix} x^2 + 1 & -4 \\ 3 - 2x^2 & x^2 + 5 \end{pmatrix}$.

A quadrilateral $Q$, whose area is 12 units, is transformed by T to $Q'$.

Find the smallest possible value of the area of $Q'$. **[5]**

**Turn over for questions 7 and 8**

**7** A transformation A is represented by the matrix $\mathbf{A}$ where $\mathbf{A} = \begin{pmatrix} -1 & x & 2 \\ 7-x & -6 & 1 \\ 5 & -5x & 2x \end{pmatrix}$.

The tetrahedron $H$ has vertices at $O$, $P$, $Q$ and $R$. The volume of $H$ is 6 units.

$P'$, $Q'$, $R'$ and $H'$ are the images of $P$, $Q$, $R$ and $H$ under A.

**(a)** In the case where $x = 5$

- find the volume of $H'$,
- determine whether A preserves the orientation of $H$. **[3]**

**(b)** Find the values of $x$ for which $O$, $P'$, $Q'$ and $R'$ are coplanar (i.e. the four points lie in the same plane). **[4]**

**8** **In this question you must show detailed reasoning.**

$\mathbf{M}$ is the matrix $\begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix}$.

Prove that $\mathbf{M}^n = \begin{pmatrix} 1 & 3(2^{n+1}-2) \\ 0 & 2^n \end{pmatrix}$, for any positive integer $n$. **[6]**

**END OF QUESTION PAPER**